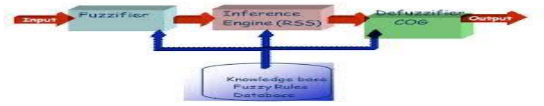


Neural Networks and Fuzzy Logic (630514)

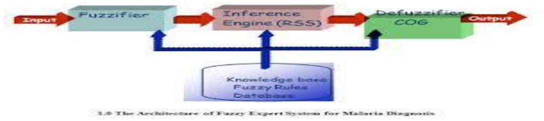
Lecture 21

Sugeno Fuzzy Models

- The main difference between ***Mamdani*** and ***Sugeno*** is that the Sugeno output membership functions are either **linear or constant**.
- Mamdani-style inference requires finding the centroid of a two-dimensional shape by integrating across a continuously varying function. In general, this process is not computationally efficient.
- **Michio Sugeno** suggested to use a single spike, a *singleton*, as the membership function of the rule consequent. A **singleton**, or more precisely a **fuzzy singleton**, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.
- The ***Sugeno Fuzzy model*** (also known as the ***TSK fuzzy model***) was proposed by Takagi, Sugeno, and Kang. A typical fuzzy rule in a Sugeno fuzzy model has the form:

$$if\ x\ is\ A\ and\ y\ is\ B\ then\ z = f(x,y)$$

  - Where **A** and **B** are fuzzy sets in the antecedent, while **z= f(x, y)** is a crisp function in the consequent.
  - Usually **f(x, y)** is a polynomial of the input variables **x** and **y**. but it can be any function.
  - When **f(x, y)** is a first-order polynomial, the resulting fuzzy inference system is called a ***first-order Sugeno fuzzy model***, for example **z = ax + by + c.**
  - When **f** is a constant, we then have a ***zero-order Sugeno fuzzy model***, **(z = ax + by + c where (a = b = 0))** which can be viewed either as a special case of the Mamdani Fuzzy inference system in which each rule's consequent is specified by a ***fuzzy singleton***.



- Each rule weights its output level, ***z_i***, by the firing strength of the rule, ***w_i***. For example, for an ***AND*** rule with **Input 1 = x** and **Input 2 = y**, the firing strength is

$$w_i = AndMethod(F_1(x), F_2(y))$$

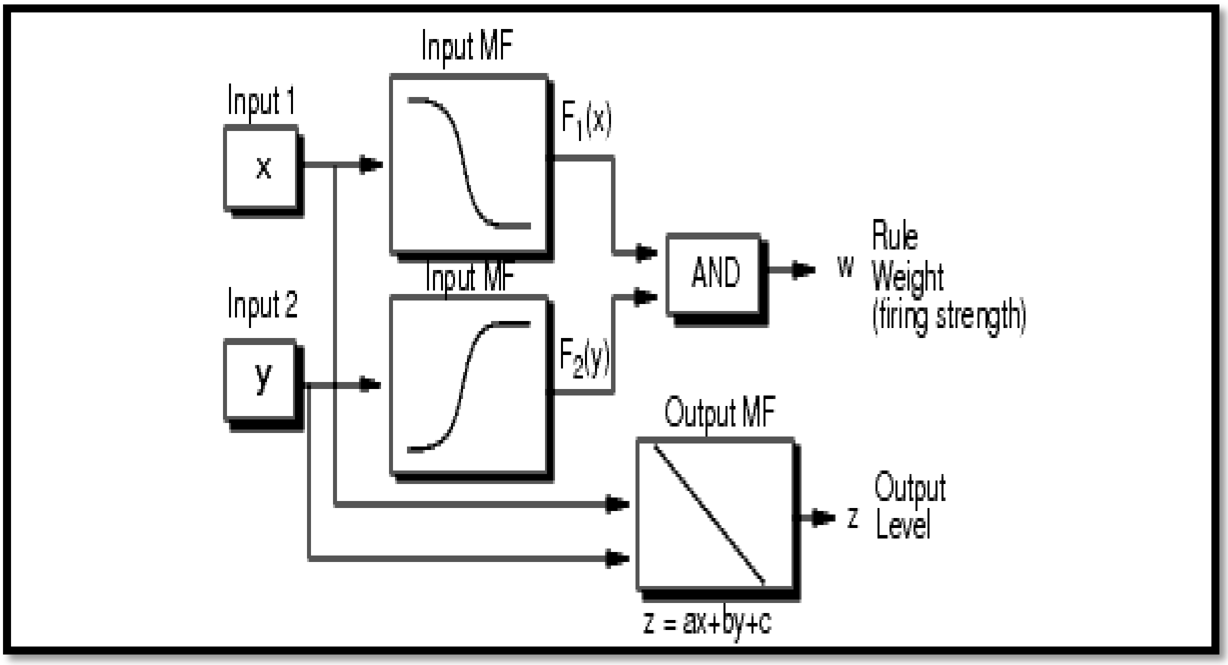
Where **F1(x), F2(y)** are the membership functions for **Inputs 1** and **2**.

- The final output of the system is the weighted average of all rule outputs, computed as

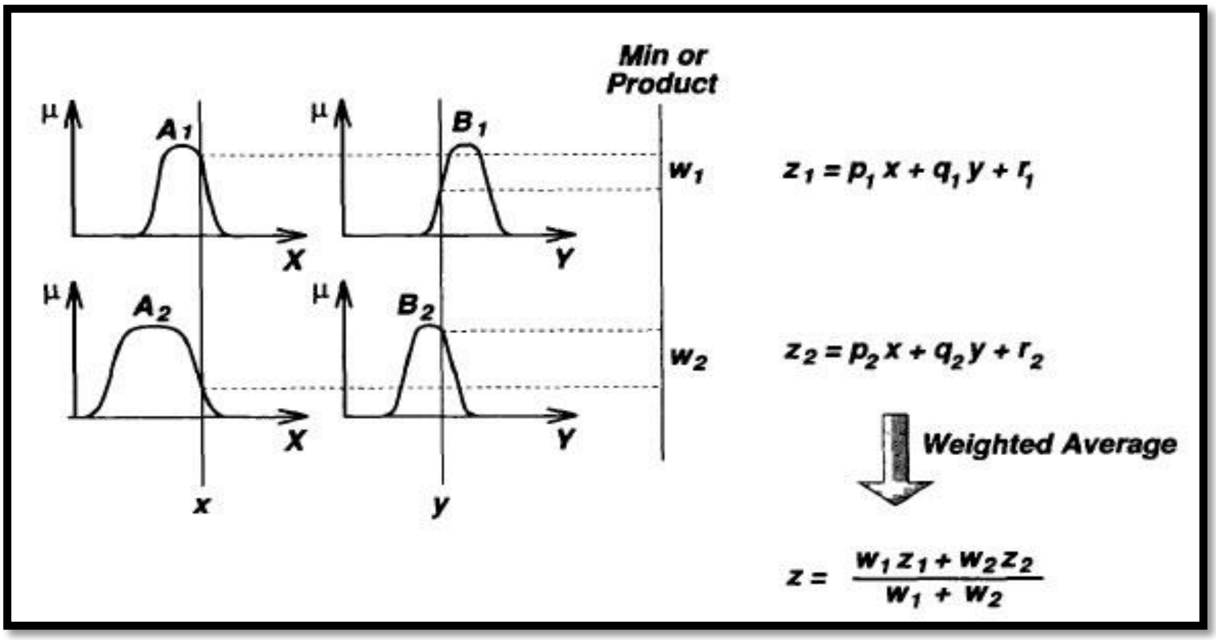
$$\text{Final Output} = \frac{\sum_{i=1}^{N} w_i z_i}{\sum_{i=1}^{N} w_i}$$

Where **N** is the number of rules

- A Sugeno rule operates as shown in the following diagram.



- Figure shows the fuzzy reasoning procedure for a first-order Sugeno fuzzy model. Since each rule has a ***crisp output***, the overall output is obtained via ***weighted average***, thus avoiding the time-consuming process of defuzzification required in a Mamdani model.

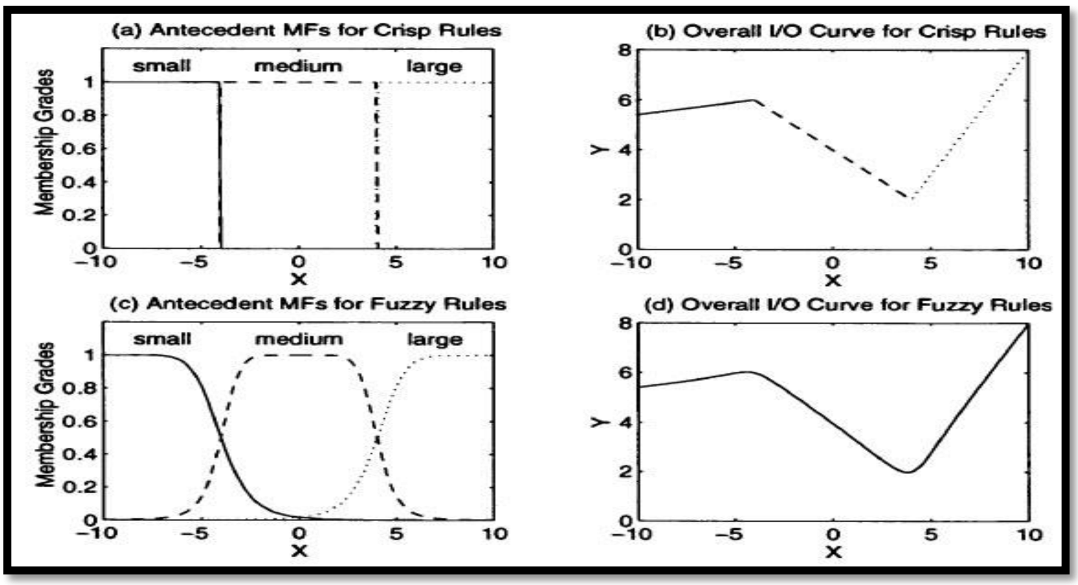


1st order Sugeno Fuzzy Model

- **Example 1: Fuzzy and nonfuzzy rule set – a comparison**

$$\begin{cases} If\ X\ is\ small\ then\ Y = 0.1X + 6.4 \\ If\ X\ is\ medium\ then\ Y = -0.5X + 4 \\ If\ X\ is\ large\ then\ Y = X - 2 \end{cases}$$

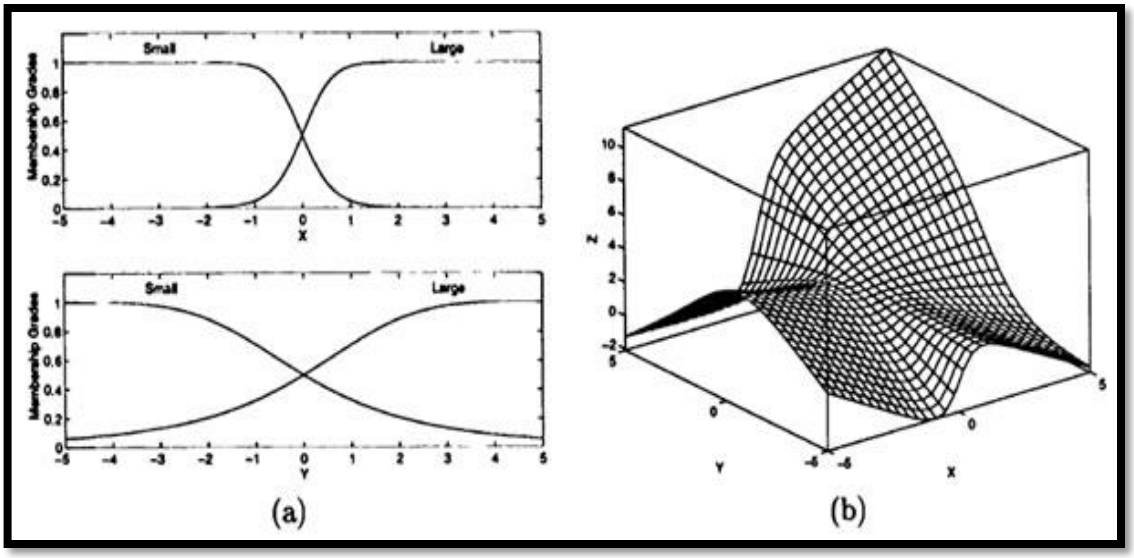
  - If "***small***", "***medium***", and "***large***" are **nonfuzzy** sets with membership functions shown Figure (a), the overall input-output curve is **piecewise linear**, as shown in Figure (b).
  - If "***small***", "***medium***", and "***large***" are **nonfuzzy** sets with membership functions shown Figure (c), the overall input-output curve becomes a smoother one, as shown in Figure (d).



- **Example 2: Two-input single-output Sugeno fuzzy model**

$$\begin{cases} If\ X\ is\ small\ and\ Y\ is\ small\ then\ z = -x + y + 1 \\ If\ X\ is\ small\ and\ Y\ is\ large\ then\ z = -y + 3 \\ If\ X\ is\ large\ and\ Y\ is\ small\ then\ z = -x + 3 \\ If\ X\ is\ large\ and\ Y\ is\ large\ then\ z = x + y + 2 \end{cases}$$

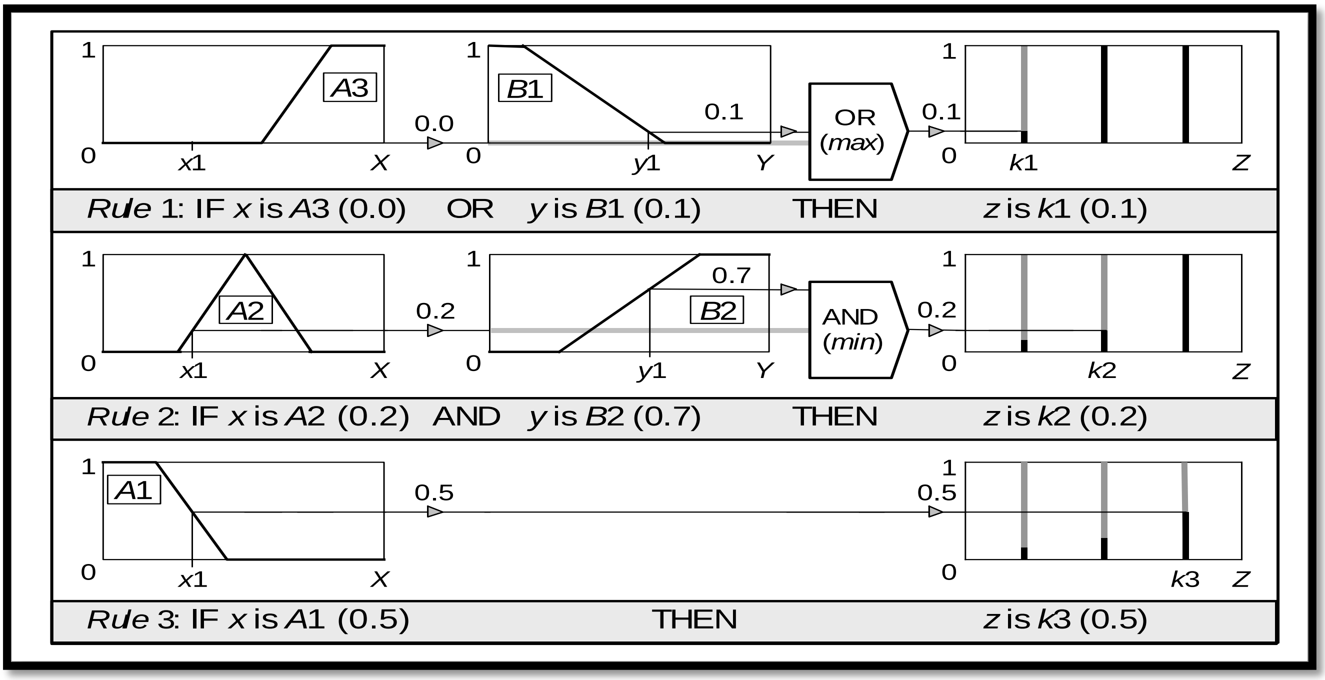
  - Figure (a) plots the membership functions of input **X** and **Y**, and Figure (b) is the resulting **input-output surface**. The surface is **complex**; the surface is composed of **four planes**, each of which is specified by output equation of a fuzzy rule.



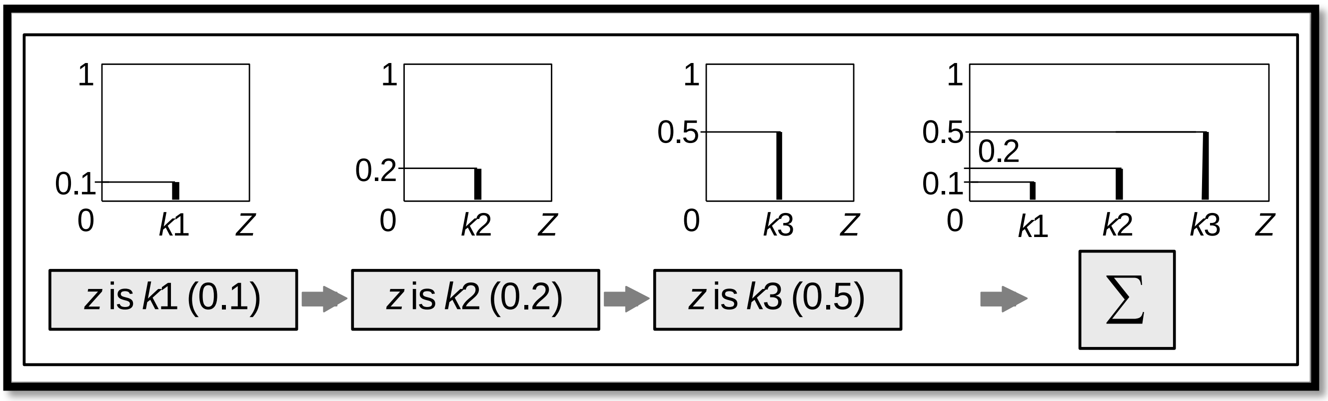
(a) (b)

Example 3:

Sugeno-style rule evaluation



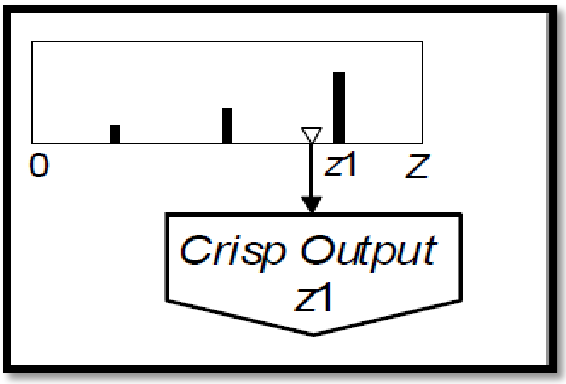
Sugeno-style aggregation of the rule outputs



Weighted average (WA):

$$WA = \frac{\mu(k1) \times k1 + \mu(k2) \times k2 + \mu(k3) \times k3}{\mu(k1) + \mu(k2) + \mu(k3)} = \frac{0.1 \times 20 + 0.2 \times 50 + 0.5 \times 80}{0.1 + 0.2 + 0.5} = 65$$

Sugeno-style defuzzification



How to make a decision on which method to apply – Mamdani or Sugeno?

- Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, Mamdani-type fuzzy inference entails a substantial computational burden.
- On the other hand, Sugeno method is computationally effective and works well with optimization and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.